Kinds Are Calling Conventions:
Intensional Static Polymorphism

Paul Downen

POPV, November 14, 2020

Theory and Practice

Of Programming Languages

Theory and Practice

Of Programming Languages

- **Goal:** Performance

Theory and Practice

Of Programming Languages

- **Goal:** Performance
- **Subgoal:** Semantics

Theory and Practice

Of Programming Languages

- **Goal:** Performance
- **Subgoal:** Semantics
- **Answer:** Logic

Compilation Funnel

Source → Intermediate → Target

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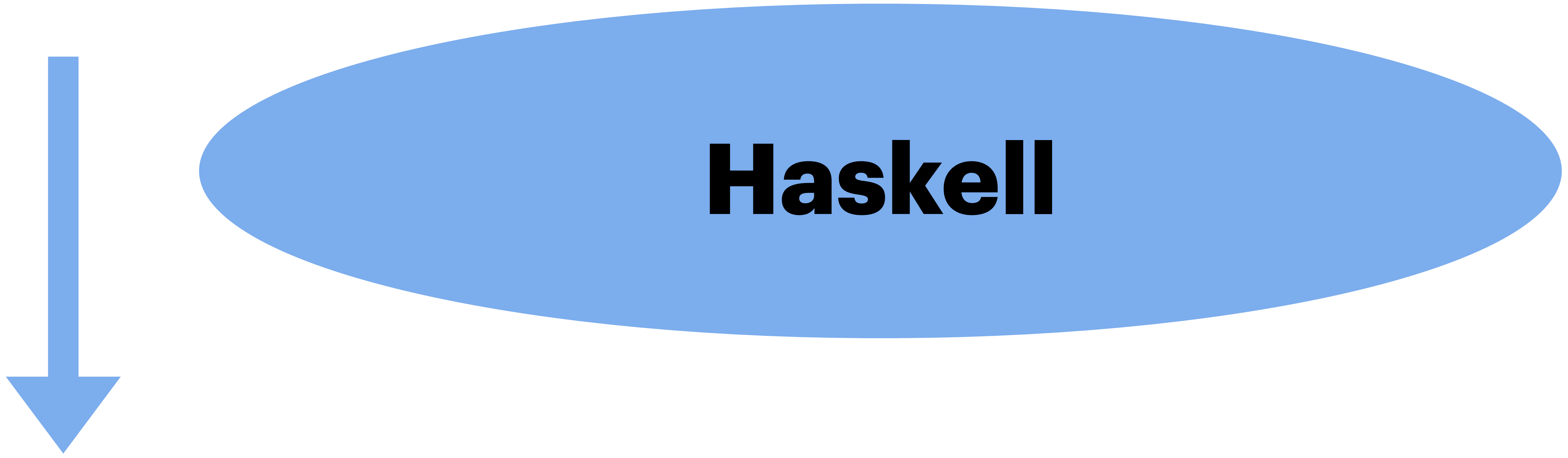
Desugaring



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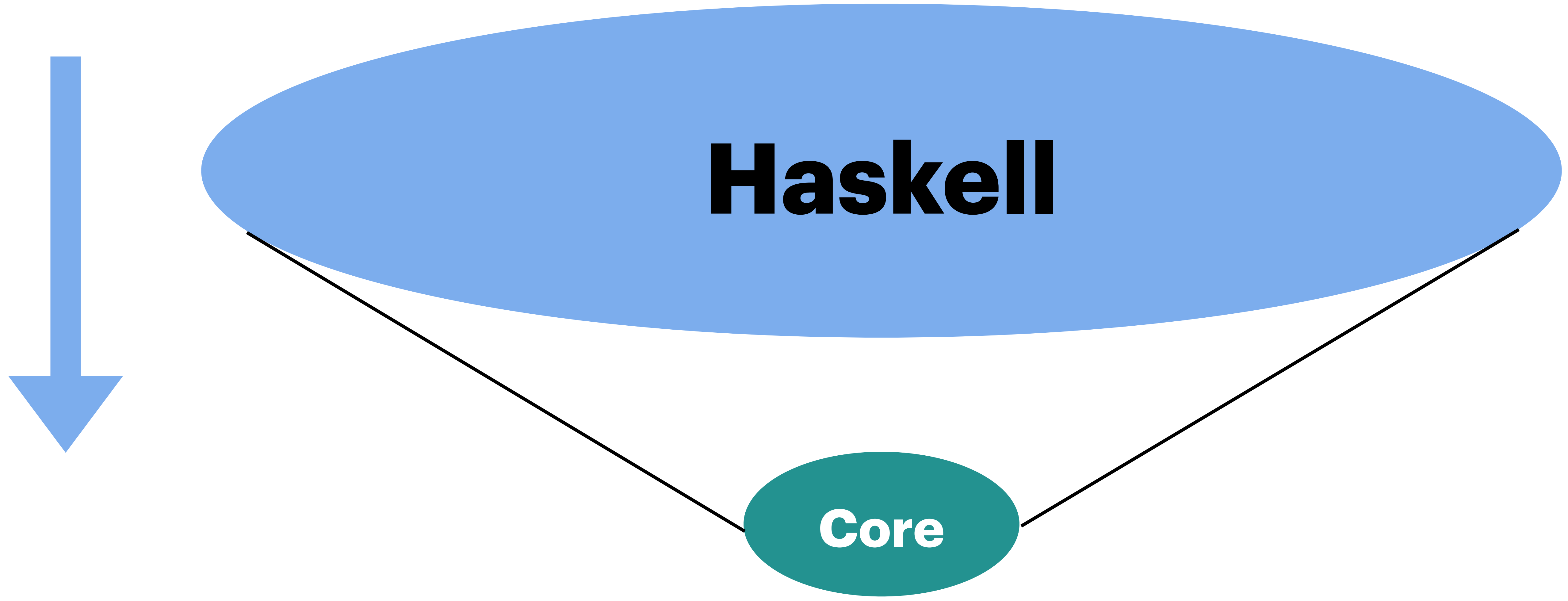

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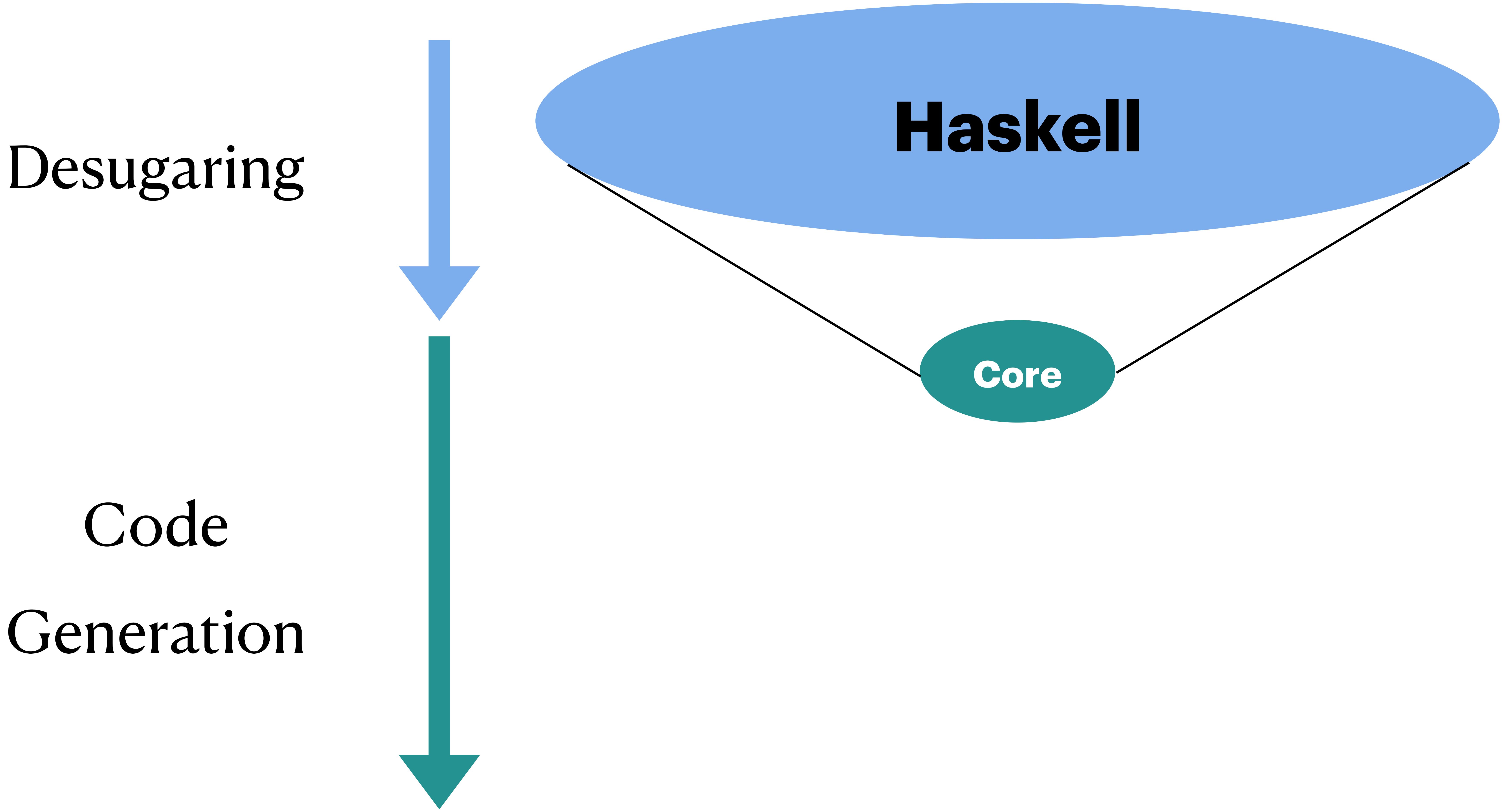

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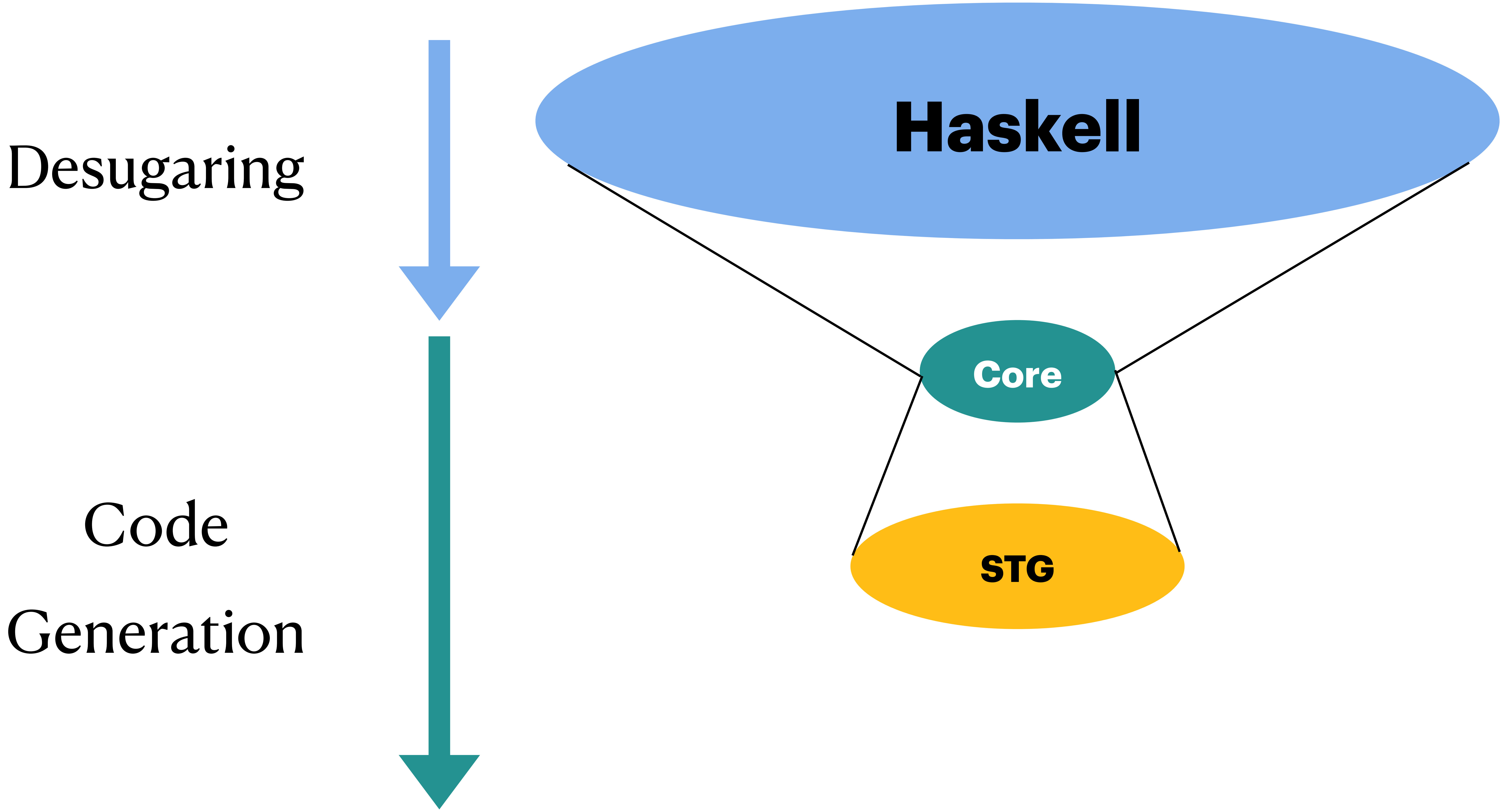

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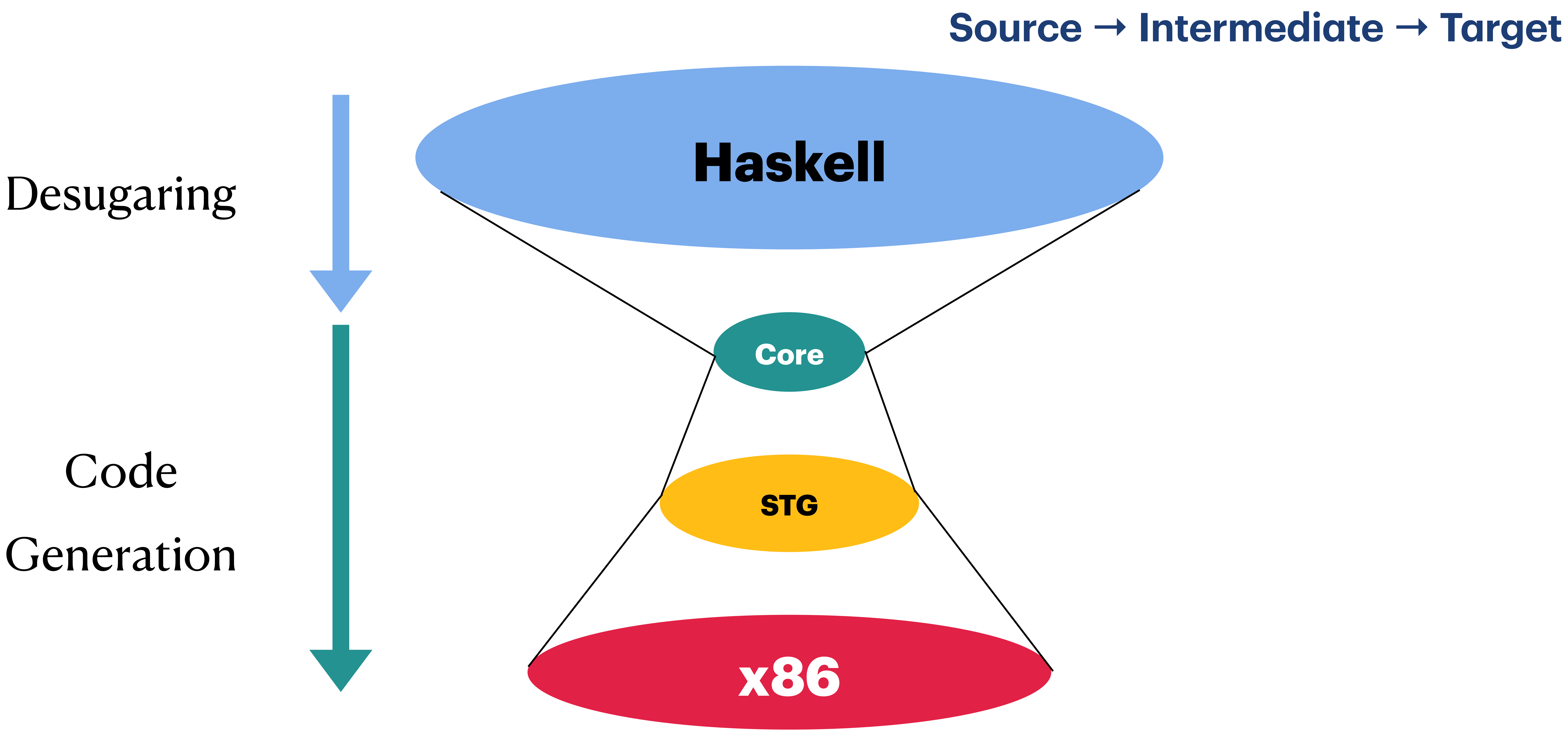

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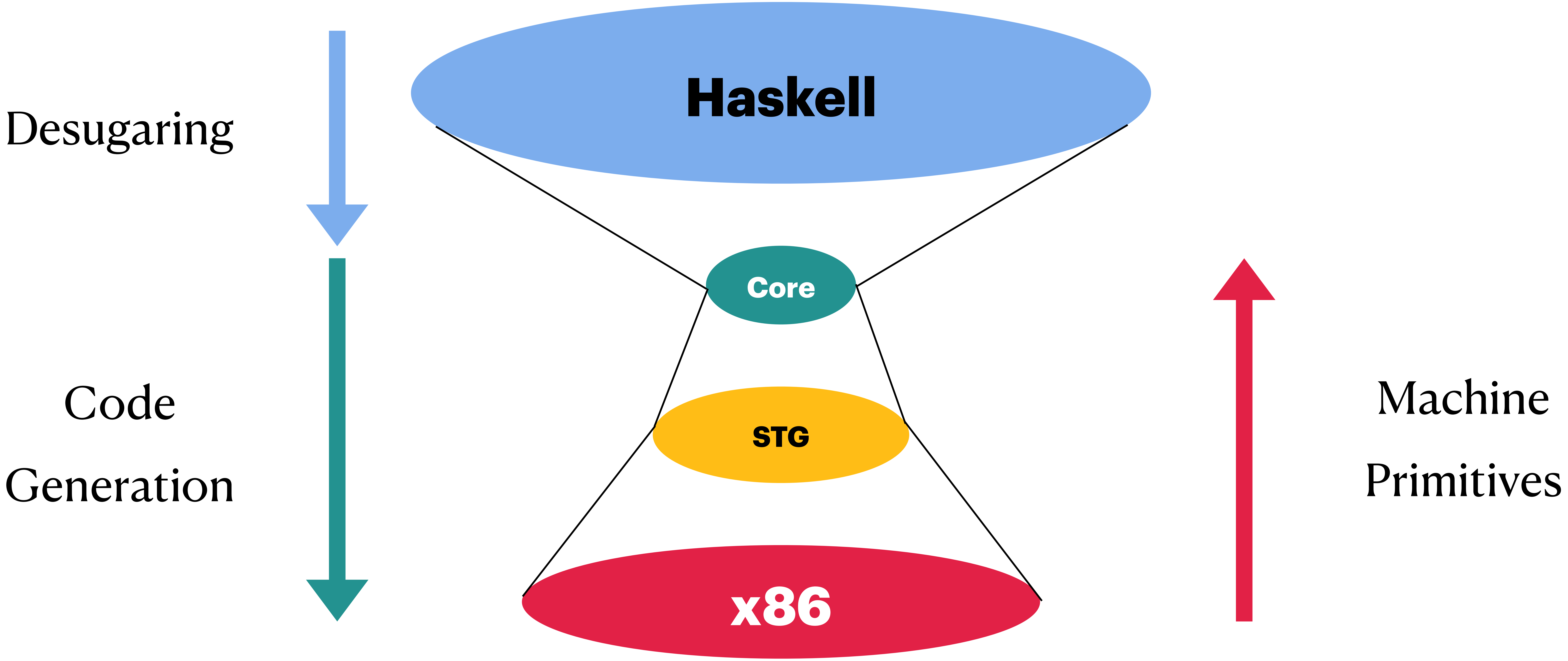

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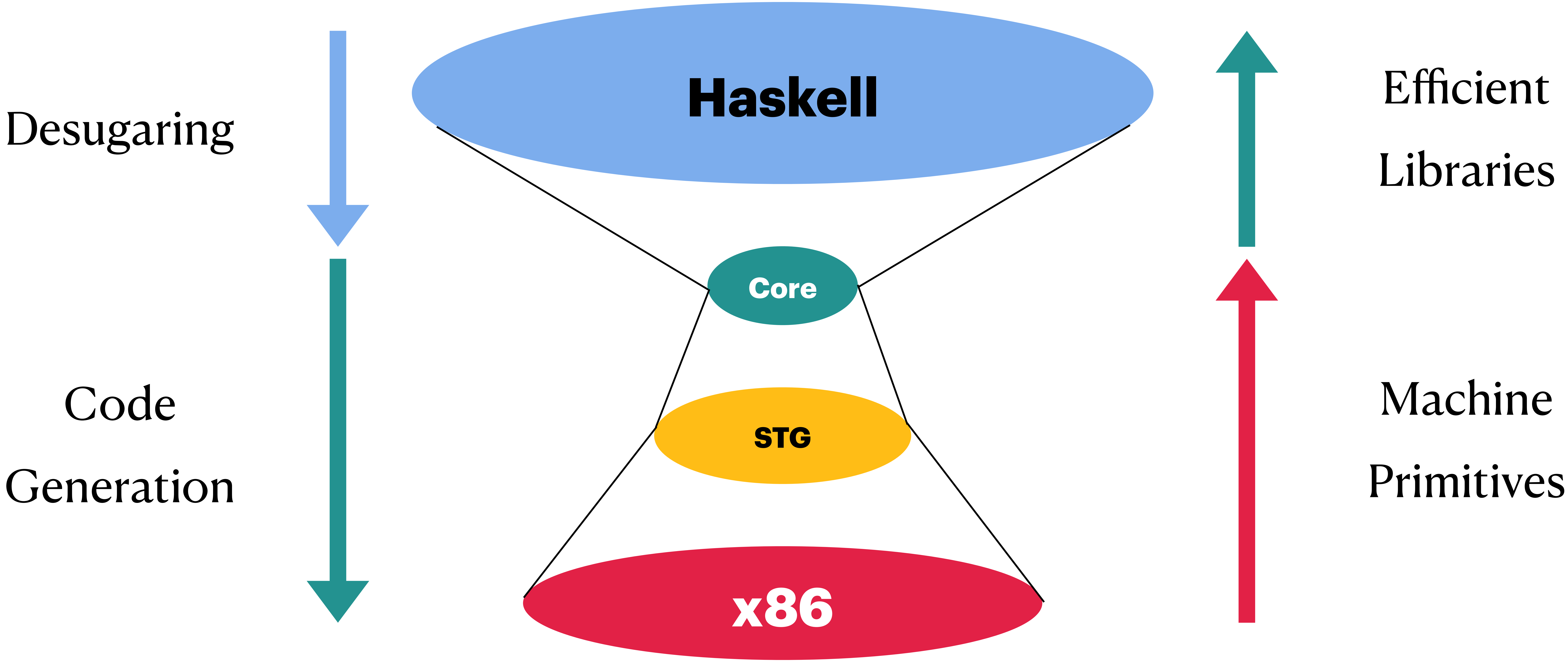

System F

Workhorse of Functional Compilers

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Core

System F

Workhorse of Functional Compilers

Core = System F

System F

Workhorse of Functional Compilers

Core = System F (first-class functions, polymorphism)

System F

Workhorse of Functional Compilers

$$\begin{aligned} \text{Core} &= \text{System F} \\ &+ \text{Data Types} \end{aligned}$$

(first-class functions, polymorphism)

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Core	=	System F	(first-class functions, polymorphism)
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Core	=	System F	(first-class functions, polymorphism)
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	+	...	

GHC Core*

*In Greek

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$Expr \ni d, e, f ::= x \mid \lambda x{:}\tau \,.\, e \mid f\ e$

λ-calculus: variables, functions, application

GHC Core*

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λ-calculus: variables, functions, application

System F: polymorphism & instantiation

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Literal primitives & let-bindings

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λ-calculus: variables, functions, application

System F: polymorphism & instantiation

Literal primitives & let-bindings

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Profiling & instrumentation

A real-world programming language in only 6 lines!

Compiling Polymorphism

Statically

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```
dup : forall a. (a -> a -> a) -> a -> a
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1. Accept parameters

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   - How many arguments does `f : a -> a -> a` take? Is `f x x : a` a call? a closure?
   - Check the arity of `f`; read runtime closure info, and take appropriate action

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In Systems Programming Languages

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- Checks for calling conventions statically at compile time

Efficient Function Calls

Parameter Passing Techniques

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- Representation — What & Where?

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Efficient Function Calls

Parameter Passing Techniques

- Representation — What & Where?
  - Shape of data values
- Arity — How many arguments?
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- Levity — When to compute?
  - Aka Evaluation Strategy
- **Goal:** A type safe high-level functional IL (System F) with fine-grained control over efficient calling conventions

The Long Road

To Intensional Static Polymorphism

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Representation

Unboxed Types

And Their Representation

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- Primitive types:

Unboxed Types

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  - `Int#`, `Float#`, `Char#`, `Word16#`, `Array#`…

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- **Pro:** Efficient memory

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`dup (++) [0..3]` — read/write pointer to `[0..3]`

versus

`dup addFloat# 1.5` — read/write float `1.5`

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Assembly code of `dup` depends on type `a`!

A Stop-Gap Solution

Uniform Polymorphism in a Nonuniform Language

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  - k may be ⋆ or ⋆->⋆ but never #
- Draconian restriction is unsatisfactory
  - **Too restrictive:** Identical definitions/code repeated for different types (like `error :: String -> a`)
  - **Incompatible with kind polymorphism:** `forall k::Kind. forall a::k. ???`

Representation Polymorphism

Kinds As Representations

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Never move or store representation-polymorphic values

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  - `(\x. … x …)` reads x
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  - `(f x)` moves (reads and writes) x

Efficient Code Abstraction

For Numeric Operations

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class Num (a              ) where
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```
data NumDict (a :: TYPE r) = NumD (a -> a -> a) …
NumFloat# = NumD addFloat# …
(+) :: forall (r :: Rep) (a :: TYPE r).
       NumDict a -> (a -> a -> a)
(+) (NumD plus …) = plus
```

Arity

Determining Function Arity

Type suggests arity 2

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Hint: ‘`expensive x`’ may be costly, or even cause side effects

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Goal: A core language with ***unrestricted*** η for functions

Static Arity

In an Intermediate Language

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Static Arity

P. Downen, Z. Sullivan, Z.M. Ariola, and S. Peyton Jones. 2019.

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And Compiling Static Polymorphism

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- How to statically compile? Is ‘`g 4`’ a call? A partial application?

Another Stop-Gap Solution

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- Wait… this sounds awfully familiar…

Arity Polymorphism

Kinds As Calling Conventions

Arity Polymorphism

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  - `a::TYPE r Call[n]` says `a` has arity n (simplified)

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P. Downen, Z.M. Ariola, S. Peyton Jones, R.A. Eisenberg. 2020.

Kinds As Calling Conventions

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Arity Polymorphism

And Higher-Order Functions

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- `f :: Int ~> Int ~> a :: TYPE Ptr Call[2+?]` has an unknown arity ≥ 2
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Restricting Arity Polymorphism

To Ensure Static Compilability

Never invoke or define arity-polymorphic code

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- When this happens in assembly depends on the compiler
- Examples:
  - `(let f = \x y z -> … in …)` defines code for `f`
  - `(\x y -> f y x)` calls code at `f`
  - `(f (\x -> …))` creates code for function pointer passed to `f`

Primitive Functions are First-Class Values

Arity-Polymorphic Data Types

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```
data List (a                 )
  = Nil | Cons a (List a)
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Nil ::
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repeat x = Cons x (repeat x)
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repeat :: forall (v :: Conv) (a :: TYPE Ptr v).
          a ~> List a
```

Efficient and Correct Abstractions

For Higher-Order Type Classes

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```
class Functor (f                                            ) where
  fmap :: (a -> b) -> f a -> f b
```

Efficient and Correct Abstractions

For Higher-Order Type Classes

```
class Functor (f :: TYPE r v -> TYPE r’ v’) where
  fmap :: (a -> b) -> f a -> f b
```

Efficient and Correct Abstractions

For Higher-Order Type Classes

```
class Functor (f :: TYPE r v -> TYPE r' v') where
  fmap :: (a -> b) -> f a -> f b

newtype Reader (e :: TYPE r v) (a :: TYPE r' v')
  = Read (e ~> a)
```

Efficient and Correct Abstractions

For Higher-Order Type Classes

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instance Functor (Reader e) where
```

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- But now `fmap id (Read g)` = `Read g!` (hint: requires η)

Efficient and Correct Abstractions

For Higher-Order Type Classes

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instance Functor (Reader e) where
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```

- But now `fmap id (Read g)` = `Read g`! (hint: requires η)
- Better for performance and correctness

Levity

Unrestricted η Is Inconsistent With Restricted β

In the λ-calculus

$$\lambda x\,.\,M\ x =_\eta M$$

Unrestricted η Is Inconsistent With Restricted β

In the λ-calculus

$$\lambda x\,.\,M\ x =_\eta M$$

$$\lambda x\,.\,\bot\ x =_\eta \bot$$

Unrestricted η Is Inconsistent With Restricted β

In the λ-calculus

$$\lambda x \,.\, M\ x =_\eta M$$

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$$(\lambda z \,.\, 5)\ (\lambda x \,.\, \bot\ x) =_\eta (\lambda z \,.\, 5)\ \bot$$

Unrestricted η Is Inconsistent With Restricted β

In the λ-calculus

$$\lambda x\,.\,M\ x =_\eta M$$

$$\lambda x\,.\perp x =_\eta \perp$$

$$(\lambda z\,.\,5)\ (\lambda x\,.\perp x) =_\eta (\lambda z\,.\,5)\ \perp$$

$$\beta_v$$

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$$(\lambda z \,.\, 5)\ (\lambda x \,.\, \bot\ x) =_\eta (\lambda z \,.\, 5)\ \bot$$

$$\downarrow \beta_v$$

$$5$$

Unrestricted η Is Inconsistent With Restricted β

In the λ-calculus

$$\lambda x\,.\,M\;x =_\eta M$$

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$$\begin{tikzcd}
(\lambda z\,.\,5)\;(\lambda x\,.\,\bot\;x) \arrow[d, "\beta_v"] & =_\eta & (\lambda z\,.\,5)\;\bot \arrow[d, "\beta_v"] \\
5 & & {}
\end{tikzcd}$$

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Unrestricted η Is Inconsistent With Restricted β

In the λ-calculus

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Goal: A core language with ***un**restricted η* for functions and *restricted β* for other types

Unboxed Data Is *Eager*

Not *Lazy*

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```
addFloat# :: Float# ~> Float# ~> Float#
```

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- Compiles to machine primop for float addition in specialized registers

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let x :: Float# = addFloat# 1.5 3.5 in …
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- Compiles to code that stores `(1.5 + 3.5)` in float register `x`

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- Can x be lazy?

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- Can x be lazy?
  - No!

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- Can x be lazy?
  - No!
  - x stores a floating-point number

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- Compiles to machine primop for float addition in specialized registers

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- Compiles to code that stores `(1.5 + 3.5)` in float register x

- Can x be lazy?
  - No!
  - x stores a floating-point number
  - Lazy thunks must be represented as pointers

Primitive Functions are *Called*

Not *Evaluated*

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```
x = let f :: Int ~> Int = expensive 100 in …f…f…
```

Primitive Functions are *Called*

Not *Evaluated*

```
x = let f :: Int ~> Int = expensive 100 in …f…f…
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- When is `expensive 100` evaluated?

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Not *Evaluated*

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  - Call-by-value: first, before binding `f`

Primitive Functions are *Called*

Not *Evaluated*

```
x = let f :: Int ~> Int = expensive 100 in …f…f…
```

- When is `expensive 100` evaluated?
  - Call-by-value: first, before binding `f`
  - Call-by-need: later, but only once, when `f` is first demanded

Primitive Functions are *Called*

Not *Evaluated*

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- `x’` always follows call-by-name order! So `x` does, too
- Primitive functions are never just *evaluated*; they are always *called*

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When Partial Application Matters

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  - map (App (f3' 100)) [1..10^6] computes 'expensive 100' only once ☺

```
Clos :: (Int ~> Int) ~> {Int ~> Int}   App  :: {Int ~> Int} ~> Int ~> Int
```

Levity and Evaluation Strategy

Denotationally and Logically

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- Logical polarity reveals the semantics for best performance

Levity Polymorphism

Call vs Eval, Revisited

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sum (I# z : xs) = case sum xs of I# y -> I# (z +# y)
```

Restricting Levity Polymorphism

To Ensure Static Compilability

Never bind or pass levity-polymorphic computations

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- Examples:
  - `(let x = expensive 100 in …)` binds x to `expensive 100`
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Code Reuse

Between Eager and Lazy Programs

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data List (                  :: TYPE Ptr v) ::
  = Nil | Cons a (List g
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```

Compilation

If it type checks, it can be compiled.

P. Downen, Z.M. Ariola, S. Peyton Jones, R.A. Eisenberg. ICFP 2020.

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To the Machine

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poly = \(f::Ptr) ~>
         let g::Ptr = \(x::Ptr, y::?, z::?) ~> f(3, x, y, z)
```

Static Compilation

With Polymorphic η-Expansion

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With Polymorphic η-Expansion

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poly :: forall a::TYPE Ptr Call[Ptr,Flt].
        (Int ~> Int ~> a) ~> (a, a)
poly f = let g :: Int ~> a = f 3
         in (g 4, g 5)
```

Static Compilation

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poly = \(f::Ptr) ~>
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```
poly = \(f::Ptr) ~>
        let g::Ptr = \(x::Ptr, y::Ptr, z::Flt) ~> f(3,x,y,z)
```

Static Compilation

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poly :: forall a::TYPE Ptr Call[Ptr,Flt].
        (Int ~> Int ~> a) ~> (a, a)
poly f = let g :: Int ~> a = f 3
         in (g 4, g 5)
```

```
poly = \(f::Ptr) ~>
        let g::Ptr = \(x::Ptr, y::Ptr, z::Flt) ~> f(3,x,y,z)
        in (\(y::Ptr, z::Flt) -> g(4, y, z),
            \(y::Ptr, z::Flt) -> g(5, y, z))
```

Lessons Learned

- *Efficient performance* requires *good semantics*
- *Good semantics* comes from *logic*
- *Kinds* capture *efficient* calling conventions

New Goal: a foundation for functional systems programming?