Controlling Copatterns

There and Back Again

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The Context

Quick Primer on Copatterns

In Scheme / Racket

Defining the 2×2 matrix

$$\text{quad} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

as the nested pair $((1, 2), (3, 4))$ by copattern matching:

```
(define*
  [((quad 'fst) 'fst) = 1]
  [((quad 'fst) 'snd) = 2]
  [((quad 'snd) 'fst) = 3]
  [((quad 'snd) 'snd) = 4])
```

Mixing Copatterns of Different Depths

Overriding the diagonals (upper left, lower right): $\left(\mathtt{diag}\ x\ y\ \begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} x & b \\ c & y \end{bmatrix}$

```
(define*
  [(((diag x y z) `fst) `fst) = x]
  [(((diag x y z) `snd) `snd) = y]
  [   (diag x y z)              = z])
```

is equivalent to (using equational reasoning) expanding the remaining options

```
(define*
  [(((diag x y z) `fst) `fst) = x]
  [(((diag x y z) `snd) `snd) = y]
  [(((diag x y z) `fst) `snd) = ((z `fst) `snd)]
  [(((diag x y z) `snd) `fst) = ((z `snd) `fst)])
```

Composable Copatterns

Combining programs along different dimensions

- Programs defined by equational reasoning on their context (à la ML, Haskell)
- Composition of extensible fragments at run-time
  - Vertical — *either or* — compose alternative options, handling failure
  - Horizontal — *and then* — compose sequence of steps, parameters, matching, guards
  - Circular — *self* — recursion back on the entire composition itself
- Side benefit: supports infinite objects, some OO-style designs

An Equational Arithmetic Evaluator

Is this ML, or Scheme?

```
; Expr = number | `(add ,Expr ,Expr) | `(mul ,Expr ,Expr)

; expr0 : Expr
(define expr0 `(add 1 (mul 2 3)))

; (arith `eval Expr) : number
(define-object
  [(arith `eval n) (try-if (number? n))
   = n]
  [(arith `eval `(add ,l ,r))
   = (+ (arith `eval l) (arith `eval r))]
  [(arith `eval `(mul ,l ,r))
   = (* (arith `eval l) (arith `eval r))])

(arith `eval expr0) = 7
```

Extending Evaluation with a New Operator

The wrong way

```
; Expr = ... | `(neg ,Expr)

; expr1 : Expr
(define expr1 `(add 1 (neg (mul 2 3))))

(define-object
  [(arith-wrong `eval `(neg ,e))
   = (- (arith-wrong `eval e))]
  [(arith-wrong `eval e) = (arith `eval e)])

(arith-wrong `eval expr1)
=
(+ 1 (arith `eval `(neg (mul 2 3))))
=/=
```

Extending Evaluation with a New Operator

The correct way, using vertical composition

```
; Expr = ... | `(neg ,Expr)

; expr1 : Expr
(define expr1 `(add 1 (neg (mul 2 3))))
```

Correct vertical composition:

```
; (arith-ext `eval Expr) : number
(define arith-ext
  (arith `compose
   (object
    [(self `eval `(neg ,e))
     = (- (self `eval e))])))

(arith-ext `eval expr1)
=
(+ 1 (arith-ext `eval `(neg (mul 2 3))))
=
-5
```

Understanding the Result of Composition

Expanding the vertical composition:

```
(define-object
  [(arith-ext ‘eval n) (try-if (number? n))
   = n]
  [(arith-ext ‘eval ‘(add ,l ,r))
   = (+ (arith-ext ‘eval l) (arith-ext ‘eval r))]
  [(arith-ext ‘eval ‘(mul ,l ,r))
   = (* (arith-ext ‘eval l) (arith-ext ‘eval r))]
  [(arith-ext ‘eval ‘(neg ,e))
   = (- (arith-ext ‘eval e))])
```

How Do We Extend Arithmetic to Algebra?

A more serious extension, needs another parameter

```
; Expr = ... | symbol

; expr2 : Expr
(define expr2 `(add x (neg (mul 2 y))))

; Env = ((symbol . number ) ...)

; env-xy : Env
(define env-xy `((x . 10) (y . 20)))

(define-object alg ...?)
```

Extending the Evaluator with an Environment!

Is this functional, or object-oriented?

```
; ((with-env Env) `env) : Env
(define (with-env dict)
  (object [(_ ‘env) = dict]))

; ((alg Env) `env)        : Env
; ((alg Env) `eval Expr) : number
(define (alg dict)
  (arith-ext ‘compose
   (with-env dict)
   (object
    [(self ‘eval x) (try-if (symbol? x))
     = (dict-ref (self ‘env) x)])))

((alg env-xy) ‘eval expr0) = 7
((alg env-xy) ‘eval expr1) = -5
((alg env-xy) ‘eval expr2) = -30
```

The Problem

The story so far

- Compositional copatterns implemented as Scheme / Racket macros
- For free: semantics as (selective) CPS
- This gives some rules for equational reasoning...

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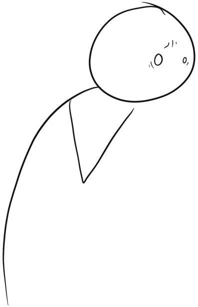
- Compositional copatterns implemented as Scheme / Racket macros
- For free: semantics as (selective) CPS
- This gives some rules for equational reasoning…
  - …but not enough to calculate every answer
- Really need a standard operational semantics
  - Reason about copattern-matching programs directly (no translation)
  - State and prove type safety problems

Operational Semantics of Compositional Copatterns

What to do, what to do…?



How the heck to deal with all the recursive generality and edge cases?!

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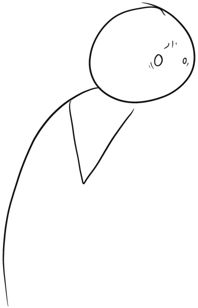

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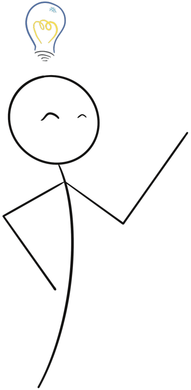

Oh wait! I remember! Olivier taught me exactly how to deal with this kind of thing!

The Unity of Semantic Artifacts

- Technique for mechanically deriving one style of semantics from another
- Using only off-the-shelf, semantics-preserving program transformations
- Correspondence between semantic artifacts for free!
- Many good intros: "A Walk in the Semantic Park"[1] is quite refreshing

[1] Olivier Danvy, Jacob Johannsen, and Ian Zerny, PEPM 2011.

The Unity of Semantic Artifacts

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- Many good intros: "A Walk in the Semantic Park"[1] is quite refreshing
- This was a key part of my second academic paper:

Classical Call-by-Need Sequent Calculi:
The Unity of Semantic Artifacts

Zena M. Ariola[1], Paul Downen[1], Hugo Herbelin[2], Keiko Nakata[3],
and Alexis Saurin[2]

Abstract. We systematically derive a classical call-by-need sequent calculus, which does not require an unbounded search for the standard redex, by using the unity of semantic artifacts proposed by Danvy *et al.* The calculus serves as an intermediate step toward the generation of an environment-based abstract machine. The resulting abstract machine is context-free, so that each step is parametric in all but one component. The context-free machine elegantly leads to an environment-based CPS

[1] Olivier Danvy, Jacob Johannsen, and Ian Zerny, PEPM 2011.

Two Roads Through the Semantic Woods

Search & Reduce

CPS

Defunctionalize

Simplify

Operational Semantics

Loop Fusion

Simplify

Deforest

Abstract Machine

Shallow patterns

Denotational style

Refunctionalize

CPS Transformation

Two Roads Through the Semantic Woods

And I had to take the one less traveled by

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The Solution

A Round-Trip in the Semantic Park

Taking the easy path to retrace our steps

- “Reforestation” and loop “fission” are hard
- Why? They undo lossy transformations
  - Loop fusion destroys the inner loops
  - Deforestation destroys intermediate data structures
- It helps to see what was lost to restore it
- Trick: First take the easy “forward” path from a similar starting point to light the way for the trip back

A Naïve Semantics for Monolithic Copatterns

Copatterns = equational reasoning on the context

$$\begin{aligned} \mathit{Term} \ni M, N &::= x \mid M\, N \mid M\, X \mid \lambda\{L \to M \ldots\} \\ \mathit{Copat} \ni \quad L &::= \varepsilon \mid x\, L \mid X\, L \end{aligned}$$

$$(\beta) \quad C[\lambda\{L_i \to M_i{}^{1 \leq i \leq n}\}] = M_j[\overrightarrow{N/x}]$$

$$\begin{pmatrix} \text{if} & C = L_j[\overrightarrow{N/x}] \\ \text{and } \forall i < j,\ \nexists \overrightarrow{N},\ & C = L_i[\overrightarrow{N/x}] \end{pmatrix}$$

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Writing an Algorithm for Small Steps

```haskell
-- Syntax
data Term      i a -- `i` represents literal index, `a` represents variable
data Copattern i a
type Question  i a = Copattern i (Term i a) -- copattern-shaped contexts
```

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-- Reduction
data Redex    i a = Respond [Option i a] | FreeVar a
data Reduct   i a = Reduced (Term i a) | Unhandled | Unknown a
data Followup i a = Next (Reduct i a) (Question i a)
                  | More (Copattern i a) (Term i a)
                         [Option i a] (Question i a)

reduce :: (Eq i, Eq a) => Redex i a -> Question i a -> Followup i a
```

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reduce :: (Eq i, Eq a) => Redex i a -> Question i a -> Followup i a

-- Search
data Found i a = Asked (Redex i a) (Question i a)

search :: Term i a -> Found i a
```

A few moments later …

The Derived CPS Transformation

Applicative forms look like normal CBN CPS:

$$
\begin{aligned}
[\![x]\!] &= x \\
[\![M\,N]\!] &= \lambda k.\ [\![M]\!]\ ([\![N]\!], k) \\
[\![M\,X]\!] &= \lambda k.\ [\![M]\!]\ (X\ k)
\end{aligned}
$$

λs begin copattern-matching various options against the continuation:

$$
\begin{aligned}
[\![\lambda\{\varepsilon\}]\!] &= \lambda k.\ k \\
[\![\lambda\{L \to M \mid L' \to M' ...\}]\!] &= \lambda k.\ [\![L \to M]\!]\ k\ [\![\lambda\{L' \to M' ...\}]\!]\ k
\end{aligned}
$$

The Derived CPS Transformation

Copatterns match on the given continuation:

$$
\begin{aligned}
[\![\varepsilon \to N]\!] &= \lambda q.\lambda f.\ [\![N]\!] \\
[\![x\ L \to N]\!] &= \mathbf{rec}\, r = \lambda q.\lambda f.\lambda k. \\
&\qquad \mathbf{case}\, k\, \mathbf{of} \begin{array}[t]{ll} (x, k') & \to [\![L \to N]\!]\ q\ f\ k' \\ () & \to r\ q\ f \\ k & \to f\ q \end{array} \\
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- If the question is too short, check again when given more continuation

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[\![\varepsilon \to N]\!] &= \lambda q.\lambda f.\, \lambda k.[\![N]\!]\; k \\
[\![x\; L \to N]\!] &= \mathbf{rec}\; r = \lambda q.\lambda f.\lambda k. \\
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\end{aligned}
$$

- If the question is too short, check again when given <u>more</u> continuation
- On successful match, keep checking the rest of the continuation
- On a mismatch failure, reset to original question and try next option

Bridging the Gap between Monolithic & Compositional

Equational reasoning to "clean up" the CPS to resemble compositional copattern macros:

- Delimiting the context
  - Add an explicit "end of message" marker ($M \,!\, R$) at the end of the question
  - Avoids confusion between "not enough context" versus "question too short"
- Nesting copatterns
  - Regroup copatterns ($L \to M$) to single steps that lean to the right (O)
  - Add an explicit "alternative" ($O \,?\, M$) for when matching fails
- Eliminate redundancy
  - Instead of passing failure+alternative+success continuations, fold failure into alternative
  - $[\![\lambda\{L \to M \mid L' \to M' \dots\}]\!] = \lambda k.\, [\![L \to M]\!]\; k\; [\![\lambda\{L' \to M' \dots\}]\!]\; k$
- Regain proper CPS through double-barrel continuations (like Shift+Reset)

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A Calculus for Compositional Copatterns with Control

$$\begin{aligned} \mathit{Response} \ni R &::= q \mid \varepsilon \mid M \,!\, R \\ \mathit{Term} \ni M, N &::= x \mid M\,N \mid M\,X \mid M. \mid \mathbf{raise} \mid O \,?\, M \mid !q \to R \\ \mathit{Option} \ni O &::= x \to O \mid X \to O \mid ?x \to M \end{aligned}$$

Old monolithic syntax now just sugar (proved correct by CPS!) on smaller primitives:

$$\begin{aligned} \lambda\{O_1 \mid \cdots \mid O_n\} &:= O_1 \,?\, (\cdots \,?\, (O_n \,?\, \mathbf{raise})) \\ \varepsilon \to M &:= ?\_ \to M \\ (x\ L) \to O &:= x \to (L \to O) \\ (X\ L) \to O &:= X \to (L \to O) \end{aligned}$$

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(x\ L) \to O &:= x \to (L \to O) \\
(X\ L) \to O &:= X \to (L \to O)
\end{aligned}
$$

But smaller primitives now give more functionality, such as vertical composition

$$
\begin{aligned}
\texttt{object}\ O &:= \lambda\{O \mid \mathit{self}\ \mathit{Open} \to \lambda\{x \to O \,?\, x\}\} \\
\mathit{compose} &:= \lambda o\ o' \to \texttt{object}\ \{?x \to o.\mathit{Open}(o'.\mathit{Open}\ x)\}
\end{aligned}
$$

$$\mathit{compose}\ \texttt{object}\{O\}\ \texttt{object}\{O'\} = \texttt{object}\{O \mid O'\}$$

A few moments later …

The Derived Operational Semantics

$$
\begin{aligned}
(?x \to N) \mathbin{?} M &\mapsto N[M/x] \\
((x \to O) \mathbin{?} M)\ N &\mapsto O[N/x] \mathbin{?} (M\ N) \\
((X \to O) \mathbin{?} M)\ X &\mapsto O \mathbin{?} (M\ X) \\
(P \mathbin{?} M)\ X &\mapsto M\ X && \text{(otherwise)} \\
(P \mathbin{?} M)\ N &\mapsto M\ N && \text{(otherwise)}
\end{aligned}
$$

$$
\begin{aligned}
E[!k \to R] \mathbin{!} \varepsilon &\mapsto R[(E[\mathbf{raise}] \mathbin{!} \varepsilon)/k] \\
M \mathbin{!} (E[\mathbf{raise}] \mathbin{!} \varepsilon) &\mapsto E[M] \mathbin{!} \varepsilon \\
(P \mathbin{?} M) \mathbin{!} \varepsilon &\mapsto M \mathbin{!} \varepsilon
\end{aligned}
$$

The Denouement

Thank You, Olivier!

- We met when I was but a green Ph.D. student
- Your generosity has taught me much
- I'm happy the influence is still alive today!

If you want to play with these toys for yourself

Semantic derivations & examples

https://github.com/pdownen/derive-copat

Copatterns for Racket & R^6RS

https://github.com/pdownen/CoScheme