Call-By-Unboxed-Value

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ICFP — Tuesday, September 3, 2023

Unboxing in Practice

The Good, the Bad, and the Ugly

- Good: Unboxed values enables high-performance
- Bad: Low-level code clashes with high-level abstractions (e.g., polymorphism)
- Representation irrelevance resolves the low-level tension (e.g., Levity Polymorphism and Kinds Are Calling Conventions)
  - Restrictions (sometimes surprising) needed for operational meaning & compilability
  - *"If I can't compile it, the type checker must reject it"*

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  - Restrictions (sometimes surprising) needed for operational meaning & compilability
  - *"If I can't compile it, the type checker must reject it"*
- Call-By-Unboxed-Value explains the high-level meaning of unboxing
  - Logical & semantic foundation ensures meaningful programs
  - *"If I can write it, I can compile & run it"*

- Compiling unboxed polymorphism before:
  - Only compile well-typed source programs; need typing information to generate code
  - Generate ill-typed target programs; compilation can break precise typing
  - *"Types describe the source, kinds describe the machine"*

Compiling With Call-By-Unboxed-Value

- Compiling unboxed polymorphism before:
  - Only compile well-typed source programs; need typing information to generate code
  - Generate ill-typed target programs; compilation can break precise typing
  - *"Types describe the source, kinds describe the machine"*
- Compiling unboxed polymorphism with Call-By-Unboxed-Value:
  - Can compile untyped source programs; no typing information needed
  - Compilation preserves typing if the source was well-typed
  - Lower-level abstract machine code can be expressed in a type-safe target language
  - Still support type erasure without changing answers

Unboxed Values

Holding Numbers in Registers

To avoid creating garbage & chasing pointers

$$\begin{aligned}
&\mathit{sumTo0} : \mathsf{Nat} \to \mathsf{Nat} \\
&\mathit{sumTo0}\ 0 = 0 \\
&\mathit{sumTo0}\ n = n + \mathit{sumTo0}(n - 1)
\end{aligned}$$

Is *n* an integer register, or a pointer into the heap?

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Is *n* an integer register, or a pointer into the heap?

Accumulator style $\implies$ fast loop

$$
\begin{aligned}
&\mathit{sumTo0'} \quad : \mathsf{Nat} \to \mathsf{Nat} \\
&\mathit{sumTo0'}\ n = \mathit{go}\ n\ 0 \\
&\qquad \textbf{where}\ \mathit{go}\ 0\ \mathit{acc} = \mathit{acc} \\
&\qquad\qquad \mathit{go}\ n\ \mathit{acc} = \mathit{go}\ (n-1)\ (n + \mathit{acc})
\end{aligned}
$$

Problems with Polymorphism

What does a compiler need to know to generate code?

Could the polymorphic *a* really be <u>any</u> type?

$$
\begin{aligned}
&id \quad : a \rightarrow a \\
&id\ x = x
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$$

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$$

Need to know a's representation to generate low-level machine code:

- Where does x live? (General or specialized register? Heap?)
- How many bits does x occupy? (32? 64? 8?)
- How to copy/move x from (incoming) parameter to (outgoing) return?

Polymorphic Ambiguity

Do we need to know *a* and *b*'s representations to compile *app*?

$$\begin{aligned} app &: (a \rightarrow b) \rightarrow a \rightarrow b \\ app\, f\ x &= f\ x \end{aligned}$$

Polymorphic Ambiguity

Do we need to know a and b's representations to compile *app*?

$$\begin{aligned} app &: (a \to b) \to a \to b \\ app\, f\ x &= f\ x \end{aligned}$$

- a: Yes, to move x
- b: It depends...
  - Naïvely yes, to move f's result to $(app\, f\ x)$'s caller
  - But with tail-call optimization, *app* never handles any b's

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What about after η-reduction?

$$\begin{aligned} app' &: (a \to b) \to a \to b \\ app'\ f &= f \end{aligned}$$

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What about after η-reduction?

$$\begin{aligned} app' &: (a \to b) \to a \to b \\ app'\ f &= f \end{aligned}$$

- a and b's representations are irrelevant!
- Only move $f : a \to b$, always a pointer

Higher-Order Ambiguity

When calling unknown functions

What do we need to know about a and b?

$$\begin{aligned} map &: (a \to b) \to [a] \to [b] \\ map\, f\, [] &= [] \\ map\, f\, (x : xs) &= (f\ x) : (map\, f\ xs) \end{aligned}$$

Higher-Order Ambiguity

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$$\begin{aligned} map &: (a \to b) \to [a] \to [b] \\ map\ f\ [] &= [] \\ map\ f\ (x : xs) &= (f\ x) : (map\ f\ xs) \end{aligned}$$

- Representations of both a and b
  - To move $x : a$ around
  - To store $(f\ x) : b$ in a list

- Calling convention of b
  - What if $f : \text{Int} \to \text{Int} \to \text{Int}$?
  - $b = \text{Int} \to \text{Int}$ is a function, needs 1 more argument
  - $(f\ x) : \text{Int} \to \text{Int}$ might be a partial application, can't jump to f's body
  - To generate code, need to distinguish partial applications from real calls

A First Taste of Call-By-Unboxed-Value

The Two Axes of Unboxing

- Familiar: Values versus computations
  - Values = being
  - Computations = doing
- New: Complexity versus Atomicity
  - Atomic = one
  - Complex = many (parts, choices, ...)

Call-By-Unboxed-Value

Having a name is a privilege, not a right

- Functions are called with complex unboxed values
    - Only atomic values are first class, can be named
    - Complex values are second class, must be matched
- Functions themselves are complex computations
    - Only atomic computations can be run directly
    - Complex computations are inert on their own, must match their context (η-long)

Elaborating Functional Code to Call-By-Unboxed-Value

Source (CBV)

$$
\begin{aligned}
&\mathit{sumTo0} \quad : \mathsf{Nat} \to \mathsf{Nat} \\
&\mathit{sumTo0}\ 0 = 0 \\
&\mathit{sumTo0}\ n = n + \mathit{sumTo0}(n - 1)
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CBPV

$$\begin{aligned} & \textit{sumTo0} : \mathsf{Nat} \to \mathsf{F}\,\mathsf{Nat} \\ & \textit{sumTo0} = \lambda n.\ \textbf{if}\ n == 0\ \textbf{then return}\ 0 \\ & \qquad\qquad\qquad\qquad\qquad \textbf{else do}\ x \leftarrow n - 1; \\ & \qquad\qquad\qquad\qquad\qquad\qquad \textbf{do}\ y \leftarrow \textit{sumTo0}\ x; \\ & \qquad\qquad\qquad\qquad\qquad\qquad n + y \end{aligned}$$

$(-) : \mathsf{Nat} \to \mathsf{Nat} \to \mathsf{F}\,\mathsf{Nat}$

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Passing & Returning Multiple Arguments

$$quotRem : \mathsf{Val\,Nat} \to \mathsf{Val\,Nat} \to \mathsf{Eval}(\mathsf{Ret}(\mathsf{Val\,Nat} \times \mathsf{Val\,Nat}))$$

Complex answers must be immediately destructed in place at the call site

OK	$\mathbf{do}\ (\mathsf{val\,int}\ q, \mathsf{val\,int}\ r) \leftarrow quotRem\ (\mathsf{val}\ 12)\ (\mathsf{val}\ 5)\ .\ \mathsf{eval}$
Illegal	$\mathbf{do}\ \mathsf{val}\ ?\ qr \leftarrow quotRem\ (\mathsf{val}\ 12)\ (\mathsf{val}\ 5)\ .\ \mathsf{eval}$

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$$distance : (\text{Val Float} \times \text{Val Float}) \to \text{Eval}(\text{Ret}(\text{Val Float}))$$

Complex arguments must be immediately constructed in place at the call site

OK	$distance\ (\text{val}\ 3.14, \text{val}\ 2.71)$
OK	$distance\ (\text{val}\ x, \text{val}\ y)$
Illegal	$distance\ xy$
Illegal	$distance\ (f\ x)$

Polymorphic Code

With type annotations...

Source	$id\ : \forall a.\ a \to a$
(System F)	$id\ = \Lambda a. \lambda(x : a).\ x$
CBUV_1	$id_1 : \forall a : \mathsf{Type}\,\mathsf{ref}\,\mathbf{val}\,.\ \mathsf{Val}\,a \to \mathsf{Eval}(\mathsf{Ret}(\mathsf{Val}\,a))$
	$id_1 = \{\,\mathsf{ty}\ a \cdot \mathsf{val}\,\mathsf{ref}(x : a) \cdot \mathsf{eval} \to \mathbf{ret}\,\mathsf{val}\,x\,\}$

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CBUV_2	$id_2 : \forall a : \mathsf{Type\,int}\ \mathbf{val}\,.\ (\mathsf{Val}\,a \times \mathsf{Val\,Float}) \to \mathsf{Eval}(\mathsf{Ret}(\mathsf{Val}\,a \times \mathsf{Val\,Float}))$
	$id_2 = \{\ \mathsf{ty}\ a \cdot (\mathsf{val\,int}(x : a), \mathsf{val\,flt}(y : \mathsf{Float})) \cdot \mathsf{eval} \to \mathbf{ret}\,(\mathsf{val}\ x, \mathsf{val}\ y)\ \}$

Polymorphic Code

With type annotations...

$$
\begin{array}{rl}
\text{Source} & id \; : \forall a.\ a \to a \\
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\text{CBUV}_2 & id_2 : \forall a : \mathsf{Type}\,\mathsf{int}\,\mathbf{val}\,.\ (\mathsf{Val}\,a \times \mathsf{Val}\,\mathsf{Float}) \to \mathsf{Eval}(\mathsf{Ret}(\mathsf{Val}\,a \times \mathsf{Val}\,\mathsf{Float})) \\
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\end{array}
$$

id_1 (Val Int × Val Float) ill-kinded, but id_1 (Box(Val Int × Val Float)) is OK because

$$\mathsf{Box} : \mathbf{cplx}\,\mathbf{val} \to \mathsf{ref}\,\mathbf{val}$$

Polymorphic Code

With type annotations...and without

$$\begin{array}{rl} \text{Source} & id \ : \forall a.\ a \to a \\ \text{(System F)} & id \ = \Lambda a.\lambda(x : a).\ x \end{array}$$

$$\begin{array}{rl} \text{CBUV}_1 & id_1 : \forall a : \text{Type ref } \mathbf{val}.\ \ \text{Val}\, a \to \text{Eval}(\text{Ret}(\text{Val}\, a)) \\ & id_1 = \{\, \text{ty}\, a \cdot \text{val ref}(x : a) \cdot \text{eval} \to \mathbf{ret}\ \text{val}\, x \,\} \end{array}$$

$$\begin{array}{rl} \text{CBUV}_2 & id_2 : \forall a : \text{Type int } \mathbf{val}.\ (\text{Val}\, a \times \text{Val Float}) \to \text{Eval}(\text{Ret}(\text{Val}\, a \times \text{Val Float})) \\ & id_2 = \{\, \text{ty}\, a \cdot (\text{val int}(x : a), \text{val flt}(y : \text{Float})) \cdot \text{eval} \to \mathbf{ret}\ (\text{val}\, x, \text{val}\, y) \,\} \end{array}$$

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$$\text{Box} : \mathbf{cplx\ val} \to \text{ref } \mathbf{val}$$

Unboxed code still has well-defined operational meaning after type erasure!

$$id_1 = \{\, \text{ty}\, a \cdot \ \text{val ref}\, x \cdot \text{eval} \to \mathbf{ret}\ \text{val}\, x \,\}$$

$$id_2 = \{\, \text{ty}\, a \cdot (\text{val int}\, x, \text{val flt}\, y) \cdot \text{eval} \to \mathbf{ret}\ (\text{val}\, x, \text{val}\, y) \,\}$$

Fusing Values and Calling Conventions

Curried & Uncurried Functions

Nested tuples & call stacks

Unboxed tuples are flattened at compile time ($a, b, c : \text{ref}\ \mathbf{val}; x : a, y : b, z : c$):

$$\begin{aligned}(\mathsf{Val}\,a \times \mathsf{Val}\,b) \times \mathsf{Val}\,c \quad &\approx \quad \mathsf{Val}\,a \times (\mathsf{Val}\,b \times \mathsf{Val}\,c) \quad \approx \quad \mathsf{Val}\,a \times \mathsf{Val}\,b \times \mathsf{Val}\,c \\ ((\mathsf{val}\,x, \mathsf{val}\,y), \mathsf{val}\,z) \quad &\approx \quad (\mathsf{val}\,x, (\mathsf{val}\,y, \mathsf{val}\,z)) \quad \approx \quad \mathsf{val}\,x, \mathsf{val}\,y, \mathsf{val}\,z\end{aligned}$$

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(Un)Curried functions are compiled to the same code ($a, b : \text{ref}\ \mathbf{val}$; $c : \text{sub}\ \mathbf{comp}$):

$$f : (\mathsf{Val}\,a \times \mathsf{Val}\,b) \to \mathsf{Eval}\,c \quad\approx\quad g : \mathsf{Val}\,a \to (\mathsf{Val}\,b \to \mathsf{Eval}\,c)$$
$$f = \{\,(\mathsf{val}\,\mathsf{ref}\,x, \mathsf{val}\,\mathsf{ref}\,y) \cdot \mathsf{eval} \to \ldots\} \quad\approx\quad g = \{\,\mathsf{val}\,\mathsf{ref}\,x \cdot (\mathsf{val}\,\mathsf{ref}\,y \cdot \mathsf{eval}) \to \ldots\}$$

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Safe due to second-class status of complex values & computations

OK	$f\ (\mathsf{val}\,x, \mathsf{val}\,y)\ .\ \mathsf{eval}$	$\approx$	$g\ (\mathsf{val}\,x)\ (\mathsf{val}\,y)\ .\ \mathsf{eval}$	OK
Illegal	$f\ xy\ .\ \mathsf{eval}$	$\not\approx$	$h\ (g\ (\mathsf{val}\,x))$	Illegal

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Unboxed tuples are flattened at compile time (a, b, c : ref **val**; $x : a, y : b, z : c$):

$$(\mathsf{Val}\ a \times \mathsf{Val}\ b) \times \mathsf{Val}\ c \quad \approx \quad \mathsf{Val}\ a \times (\mathsf{Val}\ b \times \mathsf{Val}\ c) \quad \approx \quad \mathsf{Val}\ a \times \mathsf{Val}\ b \times \mathsf{Val}\ c$$

$$((\mathsf{val}\ x, \mathsf{val}\ y), \mathsf{val}\ z) \quad \approx \quad (\mathsf{val}\ x, (\mathsf{val}\ y, \mathsf{val}\ z)) \quad \approx \quad \mathsf{val}\ x, \mathsf{val}\ y, \mathsf{val}\ z$$

(Un)Curried functions are compiled to the same code (a, b : ref **val**; c : sub **comp**):

$$f : (\mathsf{Val}\ a \times \mathsf{Val}\ b) \to \mathsf{Eval}\ c \quad \approx \quad g : \mathsf{Val}\ a \to (\mathsf{Val}\ b \to \mathsf{Eval}\ c)$$

$$f = \{\, (\mathsf{val}\ \mathsf{ref}\ x, \mathsf{val}\ \mathsf{ref}\ y) \cdot \mathsf{eval} \to \ldots \} \quad \approx \quad g = \{\, \mathsf{val}\ \mathsf{ref}\ x \cdot (\mathsf{val}\ \mathsf{ref}\ y \cdot \mathsf{eval}) \to \ldots \}$$

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Illegal	$f\ xy\ .\ \mathsf{eval}$	$\not\approx$	$h\ (g\ (\mathsf{val}\ x))$	Illegal
OK	**unbox** $(\mathsf{val}\ \mathsf{ref}\ x, \mathsf{val}\ \mathsf{ref}\ y) \leftarrow xy;$ $f\ (\mathsf{val}\ x, \mathsf{val}\ y)\ .\ \mathsf{eval}$	$\approx$	$h\ (\mathsf{clos}\{\mathsf{val}\ \mathsf{ref}\ y \cdot \mathsf{eval} \to$ $g\ (\mathsf{val}\ x)\ (\mathsf{val}\ y)\ .\ \mathsf{eval}\})$	OK

Unboxed Sums

Invariant: all complex patterns can be fully enumerated at compile time

Unboxed sums are also flattened at compile time (a, b, c : ref **val**; $x : a, y : b, z : c$):

$$\begin{array}{rcll}
(\mathsf{Val}\,a + \mathsf{Val}\,b) + \mathsf{Val}\,c & \approx & \mathsf{Val}\,a + (\mathsf{Val}\,b + \mathsf{Val}\,c) & \\
(0, (0, \mathsf{val}\,x)) & \approx & (0, \mathsf{val}\,x) & \text{Choice \#0} \\
(0, (1, \mathsf{val}\,y)) & \approx & (1, (0, \mathsf{val}\,y)) & \text{Choice \#1} \\
(1, \mathsf{val}\,z) & \approx & (1, (1, \mathsf{val}\,z)) & \text{Choice \#2}
\end{array}$$

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Invariant: all complex patterns can be fully enumerated at compile time

Unboxed sums are also flattened at compile time (a, b, c : ref **val**; $x : a, y : b, z : c$):

$(\mathsf{Val}\ a + \mathsf{Val}\ b) + \mathsf{Val}\ c$	$\approx$	$\mathsf{Val}\ a + (\mathsf{Val}\ b + \mathsf{Val}\ c)$	
$(0, (0, \mathsf{val}\ x))$	$\approx$	$(0, \mathsf{val}\ x)$	Choice #0
$(0, (1, \mathsf{val}\ y))$	$\approx$	$(1, (0, \mathsf{val}\ y))$	Choice #1
$(1, \mathsf{val}\ z)$	$\approx$	$(1, (1, \mathsf{val}\ z))$	Choice #2

Unboxed tuples distribute over unboxed sums (a, b, c : ref **val**; $x : a, y : b, z : c$):

$(\mathsf{Val}\ a + \mathsf{Val}\ b) \times \mathsf{Val}\ c$	$\approx$	$(\mathsf{Val}\ a \times \mathsf{Val}\ c) + (\mathsf{Val}\ b \times \mathsf{Val}\ c)$	
$((0, \mathsf{val}\ x), \mathsf{val}\ z)$	$\approx$	$(0, (\mathsf{val}\ x, \mathsf{val}\ z))$	Choice #0
$((1, \mathsf{val}\ y), \mathsf{val}\ z)$	$\approx$	$(1, (\mathsf{val}\ y, \mathsf{val}\ z))$	Choice #1

Choice Fusion

Unboxed sum parameters ≈ higher-order products

$$\mathit{maybeAdd}\ \mathrm{Nothing}\ y = y$$
$$\mathit{maybeAdd}\ (\mathrm{Just}\ x)\ y = x + y$$

Invariant: mandatory pattern-matching on complex values

Choice Fusion

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$$maybeAdd\ (\mathsf{Just}\ x)\ y = x + y$$

Invariant: mandatory pattern-matching on complex values

Two equivalent versions (Maybe $a = 1 + a$; Nothing $= (0, ())$; Just $x = (1, x)$):

$$\begin{aligned}
&maybeAdd_1 : (1 + \mathsf{Val\,Int}) \to \mathsf{Val\,Int} \to \mathsf{Eval}(\mathsf{Ret}(\mathsf{Val\,Int}))\\
&maybeAdd_1 = \{(0, ()) \cdot (\mathsf{val\,int}\ y) \cdot \mathsf{eval} \to \mathbf{ret}\ \mathsf{val}\ y \quad \text{(Choice \#0)}\\
&\qquad (1, \mathsf{val\,int}\ x) \cdot (\mathsf{val\,int}\ y) \cdot \mathsf{eval} \to x + y \quad \text{(Choice \#1)}\}\\
&maybeAdd_2 : (\mathsf{Val\,Int} \to \mathsf{Eval}(\mathsf{Ret}(\mathsf{Val\,Int})))\ \&\ (\mathsf{Val\,Int} \to \mathsf{Val\,Int} \to \mathsf{Eval}(\mathsf{Ret}(\mathsf{Val\,Int})))\\
&maybeAdd_2 = \{0 \cdot (\mathsf{val\,int}\ y) \cdot \mathsf{eval} \to \mathbf{ret}\ \mathsf{val}\ y \quad \text{(Choice \#0)}\\
&\qquad 1 \cdot (\mathsf{val\,int}\ x) \cdot (\mathsf{val\,int}\ y) \cdot \mathsf{eval} \to x + y \quad \text{(Choice \#1)}\}
\end{aligned}$$

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maybeAdd_1 &= \{(0, ()) \cdot ((\mathsf{val\,int}\ y) \cdot \mathsf{eval}) \to \mathbf{ret}\ \mathsf{val}\ y \quad \text{(Choice \#0)}\\
&\quad\ (1, \mathsf{val\,int}\ x) \cdot ((\mathsf{val\,int}\ y) \cdot \mathsf{eval}) \to x + y \quad \text{(Choice \#1)}\}\\
maybeAdd_2 &: (\mathsf{Val\,Int} \to \mathsf{Eval}(\mathsf{Ret}(\mathsf{Val\,Int}))) \,\&\, (\mathsf{Val\,Int} \to \mathsf{Val\,Int} \to \mathsf{Eval}(\mathsf{Ret}(\mathsf{Val\,Int})))\\
maybeAdd_2 &= \{0 \cdot ((\mathsf{val\,int}\ y) \cdot \mathsf{eval}) \to \mathbf{ret}\ \mathsf{val}\ y \quad \text{(Choice \#0)}\\
&\quad\ 1 \cdot ((\mathsf{val\,int}\ x) \cdot (\mathsf{val\,int}\ y) \cdot \mathsf{eval}) \to x + y \quad \text{(Choice \#1)}\}
\end{aligned}$$

$maybeAdd_1$ takes a Maybe argument; $maybeAdd_2$ gives a product of 2 functions

Stopping Fusion

Putting complex values in a Box pauses pattern-matching.

$$
\begin{aligned}
&\mathit{maybeAdd}_3 : \mathsf{Val}(\mathsf{Box}(1 + \mathsf{Val}\,\mathsf{Int})) \to \mathsf{Val}\,\mathsf{Int} \to \mathsf{Eval}(\mathsf{Ret}(\mathsf{Val}\,\mathsf{Int}))\\
&\mathit{maybeAdd}_3 = \{\,\mathsf{val}\,\mathsf{ref}\,x \cdot \mathsf{val}\,\mathsf{int}\,y \cdot \mathsf{eval} \to\\
&\qquad\qquad \mathbf{unbox}\,x\,\mathbf{as}\,\{\\
&\qquad\qquad\qquad (0, ()) \qquad\quad \to \mathbf{ret}\,\mathsf{val}\,y\\
&\qquad\qquad\qquad (1, \mathsf{val}\,\mathsf{int}\,x) \to x + y\\
&\qquad\qquad \}\\
&\qquad \}
\end{aligned}
$$

Stopping Fusion

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$$
\begin{aligned}
&maybeAdd_3 : \mathsf{Val}(\mathsf{Box}(1 + \mathsf{Val}\,\mathsf{Int})) \to \mathsf{Val}\,\mathsf{Int} \to \mathsf{Eval}(\mathsf{Ret}(\mathsf{Val}\,\mathsf{Int}))\\
&maybeAdd_3 = \{\, \mathsf{val}\,\mathsf{ref}\;x \cdot \mathsf{val}\,\mathsf{int}\;y \cdot \mathsf{eval} \to\\
&\qquad\qquad \mathbf{unbox}\;x\;\mathbf{as}\;\{\\
&\qquad\qquad\qquad (0, ()) \qquad\quad \to \mathbf{ret}\;\mathsf{val}\;y\\
&\qquad\qquad\qquad (1, \mathsf{val}\,\mathsf{int}\;x) \to x + y\\
&\qquad\qquad \}\\
&\qquad \}
\end{aligned}
$$

$maybeAdd_3 \not\approx maybeAdd_1$ $\qquad$ $maybeAdd_3 \not\approx maybeAdd_2$

Foundations for Unboxing

The Usual Division of Types

Call-By-Push-Value

$$\mathit{ValueType} \ni A ::= A_0 \times A_1 \mid A_0 + A_1 \mid \mathsf{U}\,\underline{B}$$
$$\mathit{ComputationType} \ni \underline{B} ::= A \to \underline{B} \mid \underline{B}_0 \,\&\, \underline{B}_1 \mid \mathsf{F}\,A$$

Focusing & Polarity

$$\mathit{PositiveType} \ni P^+ ::= P_0^+ \otimes P_1^+ \mid P_0^+ \oplus P_1^+ \mid {\downarrow}Q^-$$
$$\mathit{NegativeType} \ni Q^- ::= P^+ \to Q^- \mid Q_0^- \,\&\, Q_1^- \mid {\uparrow}P^+$$

- $\mathit{Value} = \mathit{Positive}$
- $\mathit{Computation} = \mathit{Negative}$

The Usual Division of Types

A commonly-repeated refrain

Call-By-Push-Value

$$\mathit{ValueType} \ni A ::= A_0 \times A_1 \mid A_0 + A_1 \mid \mathsf{U}\,\underline{B}$$
$$\mathit{ComputationType} \ni \underline{B} ::= A \to \underline{B} \mid \underline{B}_0 \,\&\, \underline{B}_1 \mid \mathsf{F}\,A$$

Focusing & Polarity

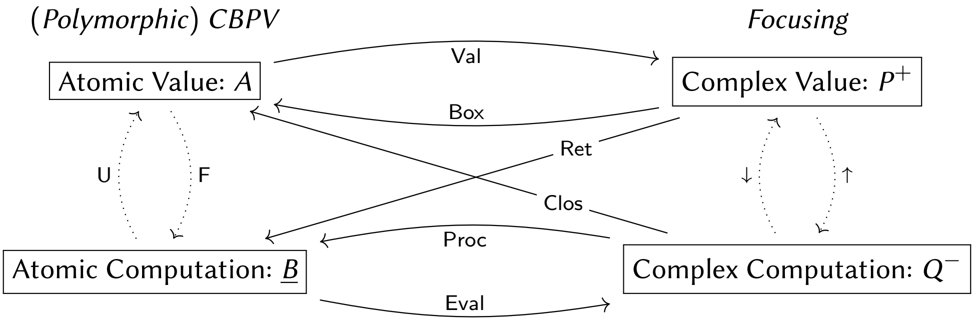
$$\mathit{PositiveType} \ni P^+ ::= P_0^+ \otimes P_1^+ \mid P_0^+ \oplus P_1^+ \mid {\downarrow}Q^-$$
$$\mathit{NegativeType} \ni Q^- ::= P^+ \to Q^- \mid Q_0^- \,\&\, Q_1^- \mid {\uparrow}P^+$$

- *Value* $=$ *Positive*?
- *Computation* $=$ *Negative*?
- Right?

A Distinction between the Distinctions

- In Call-By-Push-Value, "value types" are the denotable values
  - Only value types are first class, can be named
  - Computation types are second class, cannot be named unless "thunked"
- With strict focusing, pattern matching is mandatory
  - Positive types are second class, must be matched instead of named
  - Negative types are first class, cannot be matched so they are named
- Opposite sides of the complex vs atomic divide:
  - Call-By-Push-Value talks about atomic values and computations
  - Focusing talks about complex values and computations

Shifting Between Quadrants



$$\mathsf{F}\,A = \mathsf{Ret}(\mathsf{Val}\,A) \qquad {\uparrow}P^+ = \mathsf{Eval}(\mathsf{Ret}\,P^+)$$

$$\mathsf{U}\,\underline{B} = \mathsf{Clos}(\mathsf{Eval}\,\underline{B}) \qquad {\downarrow}Q^- = \mathsf{Val}(\mathsf{Clos}\,Q^-)$$

Equational theory: Sound & Complete w.r.t. Call-By-Push-Value!

Values = Are Computations = Do

Atomic = One Complex = Many

Higher-Order Calling Conventions

- Default "uniform" atomic representations / calling conventions:
  - Atomic value: ref = "reference" (i.e., pointer to value)
  - Atomic computation: sub = "subroutine" (i.e., return pointer)

- First-class closure values built by $\mathsf{Clos} : \mathbf{cplx\ comp} \to \mathsf{ref}\ \mathbf{val}$
  - Closure introduced by $\mathsf{clos}\{\ldots\}$ around copattern-matching code
  - Closure $f : \mathsf{Clos}\ a$ eliminated with $f.\mathsf{call}$ operation

$$app = \lambda f\ x.\ (f\ x)$$

$$app : \forall a : \mathsf{Type}\ \mathsf{ref}\ \mathbf{val}\ . \forall b : \mathsf{Type}\ \mathsf{sub}\ \mathbf{comp}\ . \downarrow(\mathsf{Val}\ a \to \mathsf{Eval}\ b) \to \mathsf{Val}\ a \to \mathsf{Eval}\ b$$

$$app = \{\mathsf{ty}\ a \cdot \mathsf{ty}\ b \cdot \mathsf{val}\ \mathsf{ref}\ f \cdot \mathsf{val}\ \mathsf{ref}\ x \cdot \mathsf{eval}\ \mathsf{sub} \to f.\mathsf{call}(\mathsf{val}\ x).\mathsf{eval}\ \mathsf{sub}\}$$

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$$app = \{\text{ty } a \cdot \text{ty } b \cdot \text{val ref } f \cdot \text{val ref } x \cdot \text{eval sub} \to f.\,\text{call}(\text{val } x).\,\text{eval sub}\}$$

$$app' = \lambda f.\, f$$

$$app' : \forall a : \text{Type } \mathbf{cplx\ val}\,.\forall b : \text{Type } \mathbf{cplx\ comp}\,.\downarrow(a \to b) \to \uparrow\downarrow(a \to b)$$

$$app' = \{\text{ty } a \cdot \text{ty } b \cdot \text{val ref } f : \text{Clos}(a \to b) \cdot \text{eval sub} \to \mathbf{ret}\ \text{val } f\}$$

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Stress Test: Representation-Polymorphic Overloading

Important Application: representation-polymorphic (type class) operator overloading

$$\begin{aligned}\textbf{class}\ \mathsf{Num}\ a\ \textbf{where}\ (+) &:: a \to a \to a \\ \mathit{negate} &:: a \to a\end{aligned}$$

What can we do without explicit representation polymorphism?

Stress Test: Representation-Polymorphic Overloading

Important Application: representation-polymorphic (type class) operator overloading

$$\textbf{class}\ \mathsf{Num}\ a\ \textbf{where}\ \begin{aligned}(+) &:: a \to a \to a \\ \mathit{negate} &:: a \to a\end{aligned}$$

What can we do without explicit representation polymorphism?

$$\textbf{type}\ \mathsf{Num}(a : \textbf{cplx val}) : \textbf{cplx val} = \mathsf{Clos}(a \to a \to {\uparrow}a) \times \mathsf{Clos}(a \to {\uparrow}a)$$

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$$\begin{aligned}(+) \quad &: \forall a : \mathsf{Type}\,\textbf{cplx val}\,.\ \mathsf{Num}\ a \to \uparrow\downarrow(a \to a \to \uparrow a) \\ (+) \quad &= \{\ \mathsf{ty}\ a \cdot (\mathsf{val\,ref}\ f, \mathsf{val\,ref}\ g) \cdot \mathsf{eval} \to \textbf{ret}\ \mathsf{val}\ f\ \}\end{aligned}$$

$$\begin{aligned}\mathit{negate} &: \forall a : \mathsf{Type}\,\textbf{cplx val}\,.\ \mathsf{Num}\ a \to \uparrow\downarrow(a \to \uparrow a) \\ \mathit{negate} &= \{\ \mathsf{ty}\ a \cdot (\mathsf{val\,ref}\ f, \mathsf{val\,ref}\ g) \cdot \mathsf{eval} \to \textbf{ret}\ \mathsf{val}\ g\ \}\end{aligned}$$

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What can we do without explicit representation polymorphism?

After type erasure, still get well-defined, operational code

$$(+) \quad = \{\ \mathsf{ty}\ a \cdot (\mathsf{val}\ \mathsf{ref}\ f, \mathsf{val}\ \mathsf{ref}\ g) \cdot \mathsf{eval} \to \textbf{ret}\ \mathsf{val}\ f\ \}$$

$$\mathit{negate} = \{\ \mathsf{ty}\ a \cdot (\mathsf{val}\ \mathsf{ref}\ f, \mathsf{val}\ \mathsf{ref}\ g) \cdot \mathsf{eval} \to \textbf{ret}\ \mathsf{val}\ g\ \}$$

Compiling to the Machine

Complex patterns $\Longrightarrow$ 1 simple switch

$$x : \mathsf{Box}((\mathsf{Val\,Int} + \mathsf{Val\,Float} \times \mathsf{Val\,Int}) + 1)$$

$$\begin{aligned}\mathbf{unbox}\,x\,\mathbf{as}\,\{\ & 0, 0, \mathsf{val\,int}\,y && \to M_1;\\ & 0, 1, \mathsf{val\,flt}\,y, \mathsf{val\,int}\,z && \to M_2;\\ & 1, () && \to M_3\}\end{aligned}$$

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```
struct {
  char tag;
  union { // case 0 = 0, 0, val int
    int zero;
          // case 1 = 0, 1, val flt, val int
    struct { float fst; int snd; } one;
          // empty case 2 = 1, ()
  } body;
} *x;
switch (x->tag) {
  case 0:
    int y = x->body.zero; M1...; break;
  case 1:
    float y = x->body.one.fst;
    int z = x->body.one.snd;
    M2...; break;
  case 2:
    M3...
}
```

Complex variables

$$\begin{aligned} \mathit{and}\ \mathsf{True}\ x &= x \\ \mathit{and}\ \mathsf{False}\ x &= \mathsf{False} \end{aligned}$$

Complex variables $x \in \{\ \mathit{pattern} \ldots \}$ match multiple patterns

$$\mathsf{Bool} = 1 + 1 \qquad \mathsf{True} = 1, () \qquad \mathsf{False} = 0, ()$$

$$\begin{aligned} &\mathit{and} : \mathsf{Bool} \to \mathsf{Bool} \to \mathsf{Eval}(\mathsf{Ret}\ \mathsf{Bool}) \\ &\mathit{and} = \{\mathsf{True} \cdot x \in \{\ \mathsf{True} \mid \mathsf{False}\ \} \cdot \mathsf{eval} \to \mathbf{ret}\ x \in \{\ \mathsf{True} \mid \mathsf{False}\ \} \\ &\qquad\quad \mathsf{False} \cdot x \in \{\ \mathsf{True} \mid \mathsf{False}\ \} \cdot \mathsf{eval} \to \mathbf{ret}\ \mathsf{False}\ \} \end{aligned}$$

is syntactic shorthand for

$$\begin{aligned} &\mathit{and} : \mathsf{Bool} \to \mathsf{Bool} \to \mathsf{Eval}(\mathsf{Ret}\ \mathsf{Bool}) \\ &\mathit{and} = \{\mathsf{True} \cdot \mathsf{True} \cdot \mathsf{eval} \to \mathbf{ret}\ \mathsf{True}; \qquad \mathsf{True} \cdot \mathsf{False} \cdot \mathsf{eval} \to \mathbf{ret}\ \mathsf{False}; \\ &\qquad\quad \mathsf{False} \cdot \mathsf{True} \cdot \mathsf{eval} \to \mathbf{ret}\ \mathsf{False}; \qquad \mathsf{False} \cdot \mathsf{False} \cdot \mathsf{eval} \to \mathbf{ret}\ \mathsf{False};\ \} \end{aligned}$$

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Complex answers

Complex continuations more $\in \{\, \textit{copattern} \ldots \}$ match multiple calling conventions

$$\begin{aligned}
\textit{app} &: \forall a : \mathsf{Type}\,\mathsf{ref}\,\mathbf{val}\,.\forall b : \mathsf{Type}\,\mathsf{sub}\,\mathbf{comp}\,.\downarrow(\mathsf{Val}\,a \to \mathsf{Eval}\,b) \to \mathsf{Val}\,a \to \mathsf{Eval}\,b \\
\textit{app} &= \{\mathsf{ty}\,a \cdot \mathsf{ty}\,b \cdot \mathsf{val}\,\mathsf{ref}\,f \cdot \mathsf{more} \in \{\,\mathsf{val}\,\mathsf{ref}\,x \cdot \mathsf{eval}\,\mathsf{sub}\,\} \to f.\,\mathsf{call}\}
\end{aligned}$$

$$\begin{aligned}
\textit{app2} &: \forall a, b : \mathsf{Type}\,\mathsf{ref}\,\mathbf{val}\,.\forall c : \mathsf{Type}\,\mathsf{sub}\,\mathbf{comp}\,. \\
&\qquad \downarrow(\mathsf{Val}\,a \to \mathsf{Val}\,b \to \mathsf{Eval}\,c) \to \mathsf{Val}\,a \to \mathsf{Val}\,b \to \mathsf{Eval}\,c \\
\textit{app2} &= \{\mathsf{ty}\,a \cdot \mathsf{ty}\,b \cdot \mathsf{ty}\,c \cdot \mathsf{val}\,\mathsf{ref}\,f \cdot \mathsf{more} \in \{\,\mathsf{val}\,\mathsf{ref}\,x \cdot \mathsf{val}\,\mathsf{ref}\,y \cdot \mathsf{eval}\,\mathsf{sub}\,\} \to f.\,\mathsf{call}\}
\end{aligned}$$

is syntactic shorthand for

$$\begin{aligned}
\textit{app} &= \{\mathsf{ty}\,a \cdot \mathsf{ty}\,b \cdot \mathsf{val}\,\mathsf{ref}\,f \cdot \mathsf{val}\,\mathsf{ref}\,x \cdot \mathsf{eval}\,\mathsf{sub} \to f.\,\mathsf{call}(\mathsf{val}\,x).\,\mathsf{eval}\,\mathsf{sub}\} \\
\textit{app2} &= \{\mathsf{ty}\,a \cdot \mathsf{ty}\,b \cdot \mathsf{ty}\,c \cdot \mathsf{val}\,\mathsf{ref}\,f \cdot \mathsf{val}\,\mathsf{ref}\,x \cdot \mathsf{val}\,\mathsf{ref}\,y \cdot \mathsf{eval}\,\mathsf{sub} \\
&\qquad\qquad \to f.\,\mathsf{call}(\mathsf{val}\,x)\;(\mathsf{val}\,y)\;.\,\mathsf{eval}\,\mathsf{sub}\}
\end{aligned}$$